2019

STATISTICS

(Major)

Paper : 2.2

(Mathematical Method—I)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following as directed : 1×7=7

(a) Define gamma integral.

(b) In a finite interval which encloses no point of infinite discontinuity, the integrand is _____ and _____.

(Fill in the blanks)

(c) When does an integral

$$\int_0^1 x^{m-1}(1-x)^{n-1}\,dx$$

exist?

(d) Define upper and lower Riemann sums of a function corresponding to the partition P.

(e) Every uniformly convergent sequence is pointwise convergent and the uniform limit function is same as the pointwise limit function.

(Write True or False)

(f) State the necessary condition for $f(x)$ to have an extreme value at the point C.

(g) If $f(xy) = 2x^2 - xy + 2y^2$, then find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ at the point (1, 2).

2. Answer the following : 2×4=8

(a) State the geometrical (physical) interpretation of Cauchy's mean value theorem.

(b) If a function f is twice derivable on $[a, a+h]$, then show that

$$f(a+h) = f(a) + hf'(a) + \frac{h^2}{2} f''(a + \theta h),$$

$0 < \theta < 1$

(c) Examine the convergence of

$$\int_0^1 \frac{dx}{\sqrt{1-x}}$$

(d) Show that

$$\int_a^b (x-a)^{l-1}(b-x)^{m-1}\,dx = (b-a)^{l+m-1}\beta(l, m)$$

3. Answer any *three* questions : 5×3=15

(a) If a function f defined on $[a, b]$ is (i) continuous on $[a, b]$, (ii) derivable on $]a, b[$ and (iii) $f(a) = f(b)$, then prove that there exists at least one real number ξ between a and b, $(a < \xi < b)$ such that $f'(\xi) = 0$.

(b) Show that the sequence $\{f_n\}$, where $f_n(x) = x^n$, is uniformly convergent on $[0, k]$ and only pointwise convergent on $[0, 1]$.

(c) If a function f defined on $[a, a+h]$ is (i) continuous on $[a, a+h]$ and (ii) derivable on $]a, a+h[$, then prove that there exists at least one number $\theta \in]0, 1[$ such that

$$f(a+h) = f(a) + hf'(a+\theta h), \quad \theta \in]0, 1[$$

(d) (i) Show that

$$\int_a^b \frac{dx}{(x-a)^P}$$

converges, if $P<1$ and diverges, if $P \geq 1$. 3

(ii) Find the value of

$$\int_0^{\pi} \cos^4 \theta \, d\theta$$ 2

(e) Show that $x^5 - 5x^4 + 5x^3 - 1$ has a maximum at $x = 1$ and minimum at $x = 3$ and neither at $x = 0$.

4. Answer any *three* questions : 10×3=30

(a) (i) State and prove Cauchy's criterion for uniform convergence. 6

(ii) Examine the validity of the hypothesis and the conclusion of Lagrange's mean value theorem for the function

$$f(x) = x(x-1)(x-2) \text{ on } \left[0, \frac{1}{2}\right]$$ 4

(b) (i) Show that

$$\frac{\Gamma(z)\Gamma(a+1)}{\Gamma(z+a)} = \sum_{n=0}^{\infty}(-1)^n \frac{a(a-1)(a-2)\ldots(a-n)}{n!}\frac{1}{z+n}$$

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(ii) Show that

$$\int_0^{\infty}\frac{x^{m-1}}{(a+bx)^{m+n}}dx = \frac{1}{a^n b^m}\beta(m, n)$$ 4

(c) (i) If α, β, γ are the roots of the equation in t, such that

$$\frac{u}{a+t}+\frac{v}{b+t}+\frac{w}{c+t}=1$$

then prove that

$$\frac{\partial(u, v, w)}{\partial(\alpha, \beta, \gamma)} = \frac{(\beta-\gamma)(\gamma-\alpha)(\alpha-\beta)}{(b-c)(c-a)(a-b)}$$ 6

(ii) Show that x^2 is integrable on any interval $[0, k]$. 4

(d) (i) Prove that the function $f(xy) = |xy|^{\frac{1}{2}}$ is not differentiable at the point (0, 0), but that f_x and f_y both exist at the origin and have the value zero. 5

(ii) A function f is bounded and integrable on $[a, b]$ and there exists a function $\phi(x)$ such that $\phi'(x) = f(x)$ on $[a, b]$, then prove that

$$\int_a^b f(x)\,dx = \phi(b) - \phi(a)$$ 5

(e) (i) Write a note on Lagrange's method of undetermined multiplier. 6

(ii) If $z = e^{xy^2}$, $x = t\cos t$, $y = t\sin t$, find $\dfrac{\partial z}{\partial t}$ at $t = \dfrac{\pi}{2}$. 4

(f) State and prove Tylor's theorem for two variables. 10

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