2019

STATISTICS

(Major)

Paper : 3.1

(Mathematical Methods—II)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following as directed : 1×7=7

(a) When is a matrix said to be an orthogonal matrix?

(b) If

$$A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

then find A^{-1}.

(c) If two matrices A and B have the same size and same rank, then which of the following is true?

(i) They have determinant zero

(ii) They are equivalent

(iii) They have common elements

(Choose the correct answer)

(d) The system of equations $AX = 0$ in n unknown has non-trivial solutions, if

(i) $\rho(A) > n$

(ii) $\rho(A) < n$

(iii) $\rho(A) = 0$

(Choose the correct answer)

(e) Write down the quadratic form for the symmetric matrix

$$\text{diag}(\lambda_1, \lambda_2, \cdots, \lambda_n)$$

(f) The rank of the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}$$

is

(i) 1

(ii) 2

(iii) 3

(Choose the correct answer)

(g) The system of equations

$$2x+3y=5,\ 6x+9y=a$$

has infinitely many solutions, if a is

(i) 2

(ii) 15

(iii) 6

(Choose the correct answer)

2. Answer the following questions : 2×4=8

(a) If A, B be n-rowed unitary matrices, then prove that AB is also a unitary matrix.

(b) Determine x, if

$$Ax=\begin{bmatrix}1\\1\\1\end{bmatrix} \text{ and } A^{-1}=\begin{bmatrix}1 & -2 & 6\\1 & -3 & 5\\-1 & 3 & -1\end{bmatrix}$$

(c) Write down the matrix of the following quadratic form :

$$x_1^2-18x_1x_2+5x_2^2$$

Also verify that they can be written as matrix products $X^T A X$.

(d) If A and B be two matrices such that AB exists, where A is non-singular, then show that AB and B have the same rank.

3. Answer any *three* of the following questions : 5×3=15

(a) If A is a non-singular matrix, then show that

$$\text{adj } (\text{adj } A) = |A|^{n-2}.A$$

(b) Interchange of a pair of rows does not change the rank. Prove it.

(c) Prove that the matrix

$$A = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} \\ \frac{-i}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{bmatrix}$$

is unitary.

(d) Prove that a necessary and sufficient condition that values, not all zero, may be assigned to the n variables

$x_1, x_2, \ldots, x_n$ so that the n homogeneous equations

$$a_{i1}x_1 + a_{i2}x_2 + \cdots + a_{in}x_n = 0,\ i = 1, 2, \cdots n;$$

hold simultaneously, is that the determinant $|a_{ij}|_{n \times n} = 0$.

(e) Find the rank of the matrix

$$A = \begin{bmatrix} 9 & 7 & 3 & 6 \\ 5 & -1 & 4 & 1 \\ 6 & 8 & 2 & 4 \end{bmatrix}$$

by reducing it to the normal form.

4. Answer any *three* of the following questions : 10×3=30

(a) Find the inverse of the matrix

$$P = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

and show that PAP^{-1} is a diagonal matrix, where A is given as

$$A = \frac{1}{2}\begin{bmatrix} b+c & c-a & b-a \\ c-b & c+a & a-b \\ b-c & a-c & a+b \end{bmatrix}$$

(b) "Every non-singular matrix is row equivalent to a unit matrix." Prove it.

(c) Find the matrices P and Q so that PAQ is of the normal form, where

$$A = \begin{bmatrix} 1 & 0 & -2 \\ 2 & 3 & -4 \\ 3 & 3 & -6 \end{bmatrix}$$

(d) "The number of linearly independent solutions of the equation $AX = 0$ is $(n-r)$, r being the rank of the $m \times n$ matrix A." Establish it.

(e) Solve completely the following system of equations :

$$x - 2y + z - w = 0$$
$$x + y - 2z + 3w = 0$$
$$4x + y - 5z + 8w = 0$$
$$5x - 7y + 2z - w = 0$$

(f) Show that the quadratic form

$$5x^2 + 26y^2 + 10z^2 + 4yz + 14zx + 6xy$$

is positive semi-definite. Also find a non-zero set of values of x, y and z which makes the form zero.

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