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STATISTICS

(Major)

Paper : 5.1

(Sampling Distribution and Statistical
Inference-I)

Full Marks : 60

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

1. Answer the following questions as directed :

1×7=7

(a) The points of inflexion of F -distribution
always exist.

(State True or False)

(b) State Cramer-Rao lower bound of the
variance of an unbiased estimator.

- (c) The degrees of freedom (df) of a chi-square statistic is 3, what will be the df of the corresponding Fisher's t -statistic?
- (d) What is asymptotic unbiasedness?
- (e) State the cumulative distribution function (c.d.f.) of the smallest order statistic $x_{(1)}$.
- (f) Generally the method of moments yields less efficient estimators than those obtained from the principle of ____.
- (Fill in the blank)
- (g) Define 'linear orthogonal transformation'.

2. Answer the following questions : 2×4=8

(a) Under what conditions, is χ^2 (chi-square) test valid?

(b) For large n , prove with usual notation

$$S.E(s^2) = \sigma^2 \times \sqrt{\frac{2}{n}}$$

(c) Find the MLE of θ in

$$f(x, \theta) = (1 + \theta) x^\theta, \quad 0 < x < 1$$

based on an independent sample of size n .

(d) A random sample of size 4 is drawn from the discrete uniform distribution

$$P(X = x) = \frac{1}{6}, \quad x = 1, 2, \dots, 6$$

Obtain the distribution function of the largest statistic.

3. Answer any *three* parts :

5×3=15

(a) Let $\hat{\theta}_n$ be an unbiased estimator of θ_n and $\text{var}(\hat{\theta}_n) = \sigma_n^2$. Also assume that

$$\left. \begin{array}{l} \theta_n \rightarrow \theta \\ \text{and } \sigma_n \rightarrow 0 \end{array} \right\} \text{ as } n \rightarrow \infty$$

Then prove that $\hat{\theta}_n$ is consistent estimator of θ .

(b) Show that the m.g.f. of $Y = \log \chi^2$, where χ^2 follows chi-square distribution with n d.f. is

$$M_Y(t) = 2^t \Gamma\left(\frac{n}{2} + t\right) / \Gamma(n/2)$$

- (c) Write down the probability function of r^2 where r is the sample correlation coefficient. If X is an F variate with 2 and $n(n \geq 2)$ degrees of freedom, then show that

$$P(X \geq K) = \left(1 + \frac{2K}{n}\right)^{-n/2}$$

- (d) Let T_1 be the MVUE of θ and T_2 be another unbiased estimator of θ , then prove that the linear combination of T_1 and T_2 will not be MVUE of θ .
- (e) Show that for t -distribution with n d.f. the mean deviation about mean is

$$\sqrt{n} \Gamma\left(\frac{n-1}{2}\right) / \Gamma\left(\frac{1}{2}\right) \Gamma\left(\frac{n}{2}\right)$$

4. Answer either (a) or (b) of the following questions :

- (a) (i) Describe briefly the method of minimum chi-square. 3
- (ii) Derive Snedecor's F -distribution. 7
- (b) (i) Find the p.d.f. of the r th order statistic $X_{(r)}$ in a random sample of size n from the exponential distribution

$$f(x) = \alpha e^{-\alpha x}, \alpha > 0, x \geq 0 \quad 4$$

(ii) Show that $X_{(r)}$ and $W_{rs} = X_{(s)} - X_{(r)}$,
are independently distributed. 4

(iii) What is the distribution of
 $W_1 = X_{(r+1)} - X_{(r)}$? 2

5. Answer either (a) or (b) of the following questions :

(a) (i) Write a brief note on the method
of moments for estimating
parameters. 3

(ii) A random variable X takes the
values 0, 1, 2 with respective
probabilities

$$\frac{\theta}{4N} + \frac{1}{2} \left(1 - \frac{\theta}{N}\right), \quad \frac{\theta}{2N} + \frac{\alpha}{2} \left(1 - \frac{\theta}{N}\right)$$

$$\text{and } \frac{\theta}{4N} + \frac{1-\alpha}{2} \left(1 - \frac{\theta}{N}\right)$$

where N is a known number and α
and θ are unknown parameters. If
75 independent observations on X
yielded the values 0, 1, 2 with
frequencies 27, 38, 10 respectively,
estimate the parameters θ and α by
the method of moments. 7

(b) (i) Let x_1, x_2, \dots, x_n be a random sample of n observations from Bernoulli population with parameter θ . Find the estimator of θ by the method of minimum chi-square. 4

(ii) If X_1 and X_2 are two independent random variables having common density function

$$f(x) = e^{-x}, \quad 0 \leq x < \infty$$

show that $u = \frac{X_1}{X_2}$ has

F -distribution with (2, 2) d.f. 6

6. Answer either (a) or (b) of the following questions :

(a) (i) State important applications of F -distribution. 3

(ii) Let the estimator of θ in $f(x, \theta)$ be T , where T is a sufficient statistic. If the MLE of θ exists, then show that it is the function of the sufficient statistic T . 7

(7)

(b) (i) State the important properties of MLE. 3

(ii) For 2×2 contingency table

a	b
c	d

prove that chi-square statistics for testing independence of attributes is given by

$$\chi^2 = \frac{N(ad - bc)^2}{(a + b)(c + d)(a + c)(b + d)}$$

where $N = a + b + c + d$. 7
