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STATISTICS

(Major)

Paper : 1.2

(Probability—I)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following as directed : 1×7=7

(a) A — is called discrete, if it contains only finitely or infinitely many sample points which can be arranged into a simple sequence.

(Fill in the blank)

(b) If A and B are two mutually exclusive events, then

(i) $P(A \cap B) = 0$

(ii) $P(\bar{A} \cap \bar{B}) = 1 - P(A) - P(B)$

(iii) $P(B) = P(A \cup B) - P(A)$

(iv) All of the above

(Choose the correct option)

(c) Given that $P(A) = \frac{1}{3}$, $P(B) = \frac{3}{4}$ and $P(A \cup B) = \frac{11}{12}$, the value of $P(B / A)$ is

(i) $\frac{1}{6}$

(ii) $\frac{4}{9}$

(iii) $\frac{1}{2}$

(iv) None of the above

(Choose the correct option)

(d) — functions are monotonically increasing.

(Fill in the blank)

(e) If X is a random variable with probability density function (p.d.f.) $f(x)$, then the mean deviation about a point $x = A$ is given by

$$MD = \int_{-\infty}^{\infty} (x - A)f(x) dx$$

(State True or False)

(f) If $M_X(t)$ is the moment generating function (m.g.f.) of X , then find the m.g.f. of $u = \frac{x-a}{h}$, where $h \neq 0$.

(g) The mathematical expectation of a random variable always exists.

(State True or False)

2. Answer the following questions : 2×4=8

- (a) State the axioms that a probability function must satisfy.
- (b) If A , B and C are three events, then write down the conditions for their mutual independence.
- (c) Define probability density function of a continuous random variable.
- (d) If X is a random variable with m.g.f.

$$M_X(t) = \frac{1}{2} e^{at+b}$$

find the value of a and b , given $E(X) = \mu$.

3. Answer any *three* of the following questions :

5×3=15

- (a) A family has three children. Let A be the event that the family has at most one girl and B be the event that not every child is of same sex. Examine whether A and B are independent events. (Assume that boys and girls are equally likely).
- (b) State and prove Bayes' theorem.
- (c) For n events A_1, A_2, \dots, A_n , show that

$$P\left(\bigcup_{i=1}^n A_i\right) \leq \sum_{i=1}^n P(A_i)$$

(d) If a random variable X assumes only positive values $0, 1, 2, 3, \dots$, then show that

$$E(X) = \sum_{n=0}^{\infty} P(X > n)$$

(e) Define distribution function. If X is a random variable with probability density function $f(x)$, find the distribution function of $Y = X^2$. Hence find the density function of Y .

4. Answer any *three* of the following questions :

10×3=30

(a) (i) Explain the classical and statistical definitions of probability. Mention the drawbacks of these definitions.

2+2+2=6

(ii) If n persons are present in a hall, what is the probability that at least two of them have a common birthday?

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(b) (i) If A and B are two events and $P(B) \neq 1$, prove that

$$P(A / \bar{B}) = \frac{P(A) - P(A \cap B)}{1 - P(B)}$$

where \bar{B} is the complementary event of B .

Hence deduce that

$$P(A \cap B) \geq P(A) + P(B) - 1 \quad 6$$

(ii) The odds in favour of a certain event are 2 to 5 and the odds against another independent event are 5 to 6. Find the chance that at least one of the events will happen.

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(c) (i) State the general law of addition of probability of n events.

In a survey report, it is stated that 65% of male college students in a city like football, 34% like hockey and 41% cricket, 10% both football and hockey, 17% both football and cricket, 15% both hockey and cricket and 6% like all three. Show that the statement is incorrect. $2+4=6$

(ii) In a bolt factory, machines A, B and C manufacture respectively 40%, 35% and 25% of the total. Of their output 5%, 4% and 3% are defective bolts. One bolt is drawn at random from the product and is found to be defective. What is the probability that it was manufactured by machine C?

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(d) (i) If X and Y are two discrete random variables, then show that

$$E\{E(X / Y)\} = E(X)$$

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- (ii) A bag contains 2^n tickets among which ${}^n C_r$ tickets bear the number r , $r = 0, 1, 2, \dots, n$. A group of m tickets are drawn. What is the mathematical expectation of the sum of their numbers? 5
- (e) (i) Define joint and marginal probability distributions.
The joint probability density function of a two-dimensional random variable (X, Y) is given by
- $$f(x, y) = 2; 0 < x < 1, 0 < y < x$$
- Examine whether X and Y are independent. 7
- (ii) A random variable X has the following probability density function :
- $$f(x) = k x(2 - x), 0 \leq x \leq 2$$
- Find k and evaluate $\text{var}(X)$. 3
- (f) (i) What are cumulants? Establish the relation between moments and cumulants. 6
- (ii) If $P(s)$ denotes the probability generating function (p.g.f.) of a random variable X , then find p.g.f. of
- (1) $P(X \leq n)$
(2) $P(X < n)$ 2+2=4

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