2016

STATISTICS

(Major)

Paper : 2.2

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions : 1×7=7

   (a) Define beta function of second kind.

   (b) Write true or false of the following statement :

   Uniform limit = Pointwise limit

   (c) What is implicit function?

   (d) Write down the gamma-duplication formula.

   (e) Define refinement.

   (f) What do you mean by extreme values of a function?

   (g) State one property of Jacobian transformation.

2. Answer the following questions : 2×4=8

(a) Define $\Gamma(z)$. Draw a rough sketch of $\Gamma(z)$.

(b) Give the geometrical interpretation of Rolle's theorem.

(c) Test the convergence of

$$\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$$

(d) If $x = r\cos\theta$, $y = r\sin\theta$, find

$$\frac{\partial(x, y)}{\partial(r, \theta)}$$

3. Answer any *three* questions : 5×3=15

(a) State and prove second mean value theorem of differential calculus. 5

(b) *(i)* If $a < b$, prove that

$$\frac{b-a}{1+b^2} < \tan^{-1} b - \tan^{-1} a < \frac{b-a}{1+a^2}$$

(ii) Show that

$$\int_0^{\pi/2} \sin^{2p} x\, dx = \int_0^{\pi/2} \sin^{2p}(2x)\, dx$$

3+2=5

(c) Find the value of

$$\int_0^{\infty} x^2 e^{-x^4} dx \times \int_0^{\infty} e^{-x^4} dx$$

5

(d) Determine the value of x, for which the function $\sin x(1+\cos x)$ attains a maximum value. 5

(e) Evaluate

$$\int_0^\infty \frac{\sin mx}{x}\,dx$$

for all real values of m. 5

4. Answer any *three* questions : 10×3=30

(a) *(i)* Find the first- and second-order partial derivatives of

$$z = x^3 + y^3 - 3axy$$

and verify that

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$$

5

(ii) Establish the relation

$$B(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}, \quad m > 0, n > 0$$

and hence deduce $\Gamma(1/2)$. 5

(b) Define functional determinant. If u, v, w are the roots of the equation in k

$$\frac{x}{a+k} + \frac{y}{b+k} + \frac{z}{c+k} = 1$$

then show that

$$\frac{\partial(x, y, z)}{\partial(u, v, w)} = -\frac{(u-v)(v-w)(w-u)}{(a-b)(b-c)(c-a)}$$

2+8=10

(c) Write a note on Lagrange's undetermined multipliers. 10

(d) (i) Prove that the necessary and sufficient conditions for the integrability of a bounded function $f(x)$ is that to every positive number ε, there corresponds a positive number δ such that for every division D whose norm is $\leq \delta$, the oscillatory sum is $< \varepsilon$. 5

(ii) If

$$\int_a^b f(x)\,dx$$

exists and k is a number such that for all x; $|f(x)| \leq k$, then prove that

$$\left|\int_a^b f(x)\,dx\right| \leq k|b-a|.$$

5

(e) (i) Discuss the continuity of the function

$$f(x, y) = \begin{cases} \dfrac{xy}{\sqrt{x^2+y^2}} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$$

at the origin. 5

(ii) State and prove Taylor's theorem for two variables. 5

(f) (i) Obtain the necessary and sufficient conditions for uniform convergence. 5

(ii) Obtain the sufficient condition for the equality of f_{xy} and f_{yx}. 5

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