

3 (Sem-3) STS M 1

2016

STATISTICS

(Major)

Paper : 3.1

(Mathematical Methods—II)

Full Marks : 60

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

1. Answer the following questions : 1×7=7

(a) Find the rank of the following matrix :

$$\begin{pmatrix} 1 & 2 & 1 & 2 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(b) Express the following quadratic form :

$$aX_1^2 + bX_2^2 + cX_3^2$$

in the form $X'AX$, where
 $X = (X_1, X_2, X_3)'$ and A is a (3×3)
matrix.

- (c) Consider the following system of homogeneous linear equations :

$$AX = 0$$

A is (3×3) , X is (3×2) and 0 is (3×1) matrix. Given $\rho(A) = 2$. Find out the number of linearly independent solutions of the system. $\rho(A)$ denotes the rank of A .

- (d) State the differences between a positive definite quadratic form and a positive semi-definite quadratic form.
- (e) Given the following system of non-homogeneous linear equations :

$$(A)_{m \times n} (X)_{n \times 1} = (B)_{m \times 1}$$

State the condition for the consistency of this system of equations.

- (f) Show that the following matrix

$$\sigma_y \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

is unitary.

- (g) Given that for a (5×5) matrix A and $|A| = 59$. Find $|3A|$.

2. Answer any *three* of the following : $5 \times 3 = 15$

(a) Show that if A is an orthogonal matrix, then A' and A^{-1} are also orthogonal.

(b) Verify whether the following system of equations is consistent or not :

$$\begin{aligned}x + y + z &= -3 \\3x + y - 2z &= -2 \\2x + 4y + 7z &= 7\end{aligned}$$

(c) Prove that the following matrix is unitary :

$$\begin{pmatrix} \frac{1+i}{2} & \frac{-1+i}{2} \\ \frac{1+i}{2} & \frac{1-i}{2} \end{pmatrix}$$

(d) Show that the multiplication of the elements of a row of a matrix by a non-zero number does not change the rank of the matrix.

(e) Obtain the matrix of the following quadratic form :

$$X_1X_2 + X_2X_3 + X_3X_1 + X_1X_4 + X_2X_4 + X_3X_4$$

3. Answer any *three* of the following : $10 \times 3 = 30$

(a) (i) Show that if the $(n \times n)$ matrix A satisfies the equation $A^2 = A$, then $\rho(A) + \rho(I - A) = n$. 6

(ii) Show that the vectors $X_1 = (1, 2, 3)'$, $X_2 = (3, -2, -1)'$ and $X_3 = (1, -6, 5)'$ form a linearly independent system. 4

(b) For what values of R , the equations

$$\begin{aligned}x + y + z &= 1 \\x + 2y + 4z &= R \\x + 4y + 10z &= R^2\end{aligned}$$

have a solution and solve them completely in each case. 10

(c) Compute the inverse of the following matrix using elementary transformation : 10

$$\begin{pmatrix} 3 & -2 & -1 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{pmatrix}$$

(d) Given

$$A = \begin{pmatrix} 2 & 6 & -2 \\ 6 & 1 & -4 \\ -2 & -4 & -3 \end{pmatrix}$$

Determine a non-singular matrix P such that $P' A P$ is a diagonal matrix. 10

(e) Prove that every $(m \times n)$ matrix of rank r can be reduced to the form $\begin{pmatrix} I_r & 0 \\ 0 & 0 \end{pmatrix}$ by a finite chain of elementary transformations, where I_r is the r -rowed identity matrix.

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4. Answer the following :

2×4=8

- (a) Given for a (3×3) matrix A , $|\text{adj} A| = 110$. Find $|A|$.
- (b) What is the difference between a sub-matrix of a matrix and its minor?
- (c) If the non-singular matrix A is symmetric, then prove that A^{-1} is also symmetric.
- (d) What is meant by the canonical form of a quadratic form? Mention one method to reduce a quadratic form to its canonical form.

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