2016

STATISTICS

(Major)

Paper : 3·2

(Distribution—I)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions : 1×7=7

(a) Write down the p.d.f. of log-normal distribution.

(b) In which distribution, mean = $\sqrt{\text{variance}}$?

(c) If $X \sim BIN\,(7, 0 \cdot 5)$, what is the mode of the distribution?

(d) Fill in the blank :

Mean deviation of normal distribution is ——.

(e) If the mean of Poisson distribution is 3, what is its variance?

(f) In which distribution

mean = median = mode?

(g) Write down the relation between mean and variance of binomial distribution.

2. Answer any *four* of the following : 2×4=8

(a) Define Weibull distribution. What are its uses?

(b) If X follows Poisson distribution such as $P(X=1)=P(X=2)$, find $P(X=4)$.

(c) State a few applications of hyper-geometric distributions.

(d) Obtain the m.g.f. of a discrete uniform distribution $\cup(1, n)$.

(e) Give an interpretation of the p.m.f. of geometric distributions.

3. Answer any *three* of the following : 5×3=15

(a) Prove that for normal distribution $\beta_1 = 0$ and $\beta_2 = 3$.

(b) Derive negative binomial distribution.

(c) If $X \sim$ expo (λ) with $P(X \leq 1) = P(X > 1)$, find $\text{var}(X)$.

(d) The p.d.f. of the random variable X is

$$f(x) = \frac{1}{\beta(m, n)} x^{m-1}(1-x)^{n-1};\ 0 \le x \le 1$$

Find the distribution of $Y = \frac{1}{X}$.

4. Answer any *three* of the following : 10×3=30

(a) Let $X \sim N(0, \sigma^2)$ and $Y \sim N(0, \sigma^2)$ be independent random variables. Show that $\frac{X}{Y}$ follows Cauchy distributions. Prove that m.g.f. of Cauchy distribution does not exist.

(b) Obtain the m.g.f. of a negative binomial distribution and show that its mean is less than its variance.

(c) Show that the point of inflexion of two-parameter gamma distribution having the p.d.f.

$$f(x) = \frac{\lambda^{\alpha}}{\Gamma\alpha} e^{-\lambda x} x^{\alpha-1};\ x > 0$$

is equidistant from the mode, provided $\alpha > 1$ and values are real and positive. Also find its mode.

(d) Show that hypergeometric distribution tends to binomial distribution under certain conditions.

(e) Prove that Poisson distribution is a limiting case of binomial distribution.

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