2017

STATISTICS

(Major)

Paper : 2.1

(Numerical and Computational Techniques—I)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following as directed : 1×7=7

(a) The value of

$$\Delta^3 (1-x)(1-2x)(1-3x)$$

with the interval of differencing being unit is

(i) 36

(ii) –36

(iii) 18

(iv) None of the above

(Choose the correct option)

(b) Which of the following relations is not correct?

(i) $\mu \equiv \frac{1}{2}[E^{1/2} + E^{-1/2}]$

(ii) $\mu^2 \equiv 1 + \frac{1}{4}\delta^2$

(iii) $\mu\delta \equiv \Delta + \nabla$

(iv) $E^{1/2} \equiv \frac{1}{2}\delta + \mu$

(Choose the correct option)

(c) Define 'inverse interpolation'.

(d) The divided differences are ____ in their arguments.

(Fill in the blank)

(e) With usual notations

$$\frac{1}{2}\delta^2 + \delta\sqrt{1 + \frac{\delta^2}{4}}$$

is equivalent to

(i) Δ

(ii) ∇

(iii) $\Delta + \nabla$

(iv) None of the above

(Choose the correct option)

(f) The tangent method of finding roots of polynomial equations is known as

(i) regula falsi method

(ii) bisection method

(iii) Newton-Raphson method

(iv) graphical method

(Choose the correct option)

(g) An equation of the form

$$u_{x+n} + p_1 u_{x+n-1} + \cdots + p_n u_x = R(x)$$

where p_i's and $R(x)$ are functions of x or constants, is called linear non-homogeneous equation of order n if $R(x) =$ ____.

(Fill in the blank)

2. Answer the following questions : 2×4=8

(a) Prove that

$$E \equiv e^{hD}$$

(Symbols have their usual meanings.)

(b) State Weddle's rule of numerical integration. Mention the underlying assumptions.

(c) Obtain the function whose first difference is $9x^2 + 11x + 5$.

(d) Solve

$$u_{x+2} - 8u_{x+1} + 15u_x = 0$$

by the method of differences.

3. Answer any *three* of the following questions : 5×3=15

(a) Show that the operators E and Δ obey the law of indices.

(b) Express the function

$$f(x) = 2x^3 - 3x^2 + 3x - 10$$

in factorial notation and hence show that $\Delta^3 f(x) = 12$.

(c) Suppose a, b, c and d are successive entries corresponding to equidistant arguments in a table. Show that the entry corresponding to the argument halfway between the arguments for b and c is

$$\frac{9(b+c)-(a+d)}{16}$$

The third difference being constant.

(d) Show that Newton-Raphson method is quadratic convergent.

(e) State and prove Bessel's central difference formula.

4. Answer any *three* of the following questions : 10×3=30

(a) (i) If u_x is a polynomial of degree n, prove that

$$\Delta^r u_x = \text{constant}, \quad r = n$$
$$= 0, \quad r > n$$

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(ii) Show that

$$u_0 + \frac{u_1 x}{1!} + \frac{u_2 x^2}{2!} + \frac{u_3 x^3}{3!} + \cdots = e^x \left[u_0 + x\Delta u_0 + \frac{x^2}{2!}\Delta^2 u_0 + \cdots \right]$$

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(b) Solve the difference equations : 5+5=10

(i) $u_{x+2} - 2mu_{x+1} + (m^2 + n^2)u_x = m^x$

(ii) $u_x - u_{x-1} + 2u_{x-2} = x + 2^x$

(c) Derive general quadrature formula for equidistant ordinates. Hence derive Simpson's one-third rule for numerical integration. 6+4=10

(d) Derive Euler-Maclaurin summation formula. 10

(e) From the following table of values of x and y, obtain $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for $x = 1 \cdot 2$: 10

x	:	1·0	1·2	1·4	1·6
y	:	2·7183	3·3201	4·0552	4·9530

x	:	1·8	2·0	2·2
y	:	6·0496	7·3891	9·0250

(f) *(i)* Explain the bisection method for obtaining the roots of the equation

$$f(x) = 0$$

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(ii) Solve

$$x^4 - x - 10 = 0$$

by Newton-Raphson method, root being near $x = 2$. 5

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