# 2017

## STATISTICS

( Major )

Paper : 2.2

**( Mathematical Methods—I )**

*Full Marks* : 60

*Time* : 3 hours

*The figures in the margin indicate full marks for the questions*

**1.** Answer the following questions : 1×7=7

(a) Define uniform convergence.

(b) What do you mean by improper integrals?

(c) State Rolle's theorem.

(d) Define the continuity of a function $f(x, y)$ at a point $(a, b)$.

(e) Define Beta function of first kind.

(f) What is Riemann integration?

(g) Define Jacobian.

**2.** Answer the following questions : 2×4=8

*(a)* State Cauchy's mean value theorem.

*(b)* State Taylor's theorem for two variables.

*(c)* Investigate the continuity of

$$f(x, y) = \begin{cases} \dfrac{x^2 - y^2}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$

at (0, 0).

*(d)* Find the value of

$$\beta\left(\frac{1}{2}, \frac{1}{2}\right)$$

**3.** Answer any *three* questions : 5×3=15

*(a)* Find the point of maxima and minima of

$$f(x) = 3\cos^2 x + \sin^6 x, -\frac{\pi}{2} < x < \frac{\pi}{2}$$

*(b)* If

$$x = u - v + w$$
$$y = u^2 - v^2 - w^2$$
$$z = u^3 + v$$

then evaluate the Jacobian

$$\frac{\partial(x, y, z)}{\partial(u, v, w)}$$

*(c)* Prove that

$$\int_0^\infty \frac{\sin x}{x}\,dx$$

converges.

*(d)* Prove that

$$\beta(m, n) = \beta(n, m)$$

*(e)* Given

$$\int_0^\infty \frac{x^{p-1}}{1+x}\,dx = \frac{\pi}{\sin p\pi}$$

Show that $\Gamma p \Gamma(1-p) = \frac{\pi}{\sin p\pi}$, when $0 < p < 1$.

**4.** Answer any *three* questions : 10×3=30

*(a)* State and prove Lagrange's mean value theorem and give the geometrical interpretation.

*(b)* Prove

$$\int_0^{\pi/2} \sin^p \theta\, d\theta = \int_0^{\pi/2} \cos^p \theta\, d\theta =$$

$$\begin{cases} \frac{1.3.5.\cdots(p-1)}{2.4.6.\cdots p}\,\frac{\pi}{2}, & \text{if } p \text{ is an even positve integer} \\ \frac{2.4.6.\cdots(p-1)}{1.3.5.\cdots p}, & \text{if } p \text{ is an odd positve integer} \end{cases}$$

(c) (i) State and prove Cauchy's criterion for uniform convergence.

(ii) State and prove Taylor's expansion for one variable.

(d) Given

$$f(x, y) = \begin{cases} \dfrac{xy(x^2 - y^2)}{(x^2 + y^2)} & , \quad (x, y) \neq (0, 0) \\ 0 & , \quad (x, y) = (0, 0) \end{cases}$$

Compute $f_x(0, 0)$, $f_{xx}(0, 0)$, $f_{yy}(0, 0)$, $f_{xy}(0, 0)$, $f_{yx}(0, 0)$.

(e) (i) With reference to the function of two variables, show that every differentiable function is continuous.

(ii) If

$$x = r\sin\theta\cos\phi$$
$$y = r\sin\theta\sin\phi$$
$$z = r\cos\theta$$

then show that

$$\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)} = r^2\sin\theta$$

(f) State and prove the necessary and sufficient condition for uniform convergence of a series.

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