2017

STATISTICS

(Major)

Paper : 3·2

(Distribution—I)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions : $1 \times 7 = 7$

(a) Under what condition, the binomial distribution possesses the additive property?

(b) What is the relation between mean and variance of geometric distribution?

(c) Give an example in which negative binomial distribution occurs.

(d) Under what conditions hypergeometric distribution tends to binomial distribution?

(e) Write down the probability density function of Weibull distribution.

(f) Name the distribution for which mean does not exist.

(g) Under what condition mean and variance of exponential distribution are equal?

2. Answer the following questions : 2×4=8

(a) If X and Y are independent Poisson variates with means λ_1 and λ_2 respectively, find the probability that $X = Y$.

(b) Derive moment generating function of geometric distribution.

(c) If X is a random variable with a continuous distribution function F, then prove that $F(X)$ has a uniform distribution on [0, 1].

(d) If X follows standard Cauchy distribution, find a p.d.f. for X^2 and identify its distribution.

3. Answer any *three* of the following : 5×3=15

(a) If X has negative binomial distribution with parameters (n, P), prove that $M_X(t) = (Q - Pe^t)^{-n}$. Hence find m.g.f. of

$$z = \frac{X - nP}{\sqrt{nPQ}}$$

and deduce that z is asymptotically normal as $n \to \infty$.

(b) Show that for a normal distribution with mean μ and variance σ^2, the central moments satisfy the relation

$$\mu_{2n} = (2n-1)\mu_{2n-2}\sigma^2$$

(c) X_1 and X_2 are two independent random variables possessing same geometric distribution. Then show that

$$P(X_1 | X_1 + X_2 = n) = \frac{1}{n+1}; n = 0, 1, 2, \cdots$$

(d) If X and Y are independent gamma variates with parameters m and n respectively, show that the variables $U = X + Y$ and $V = \frac{X}{X+Y}$ are independently distributed and identify their distributions.

(e) If X follows Laplace distribution with parameters λ and μ, find the rth moment about origin.

4. Answer any *three* of the following : 10×3=30

(a) Let the random variable X be distributed uniformly on $(0, 1)$ with p.d.f. $f(x) = 1, \quad 0 \le x \le 1$. Assume that the conditional distribution of Y given $X = x$ has a binomial distribution with p.m.f.

$$P(Y = y | X = x) = \binom{n}{y} x^y (1-x)^{n-y};$$

$$y = 0, 1, 2, \cdots, n$$

where n and x are the parameters of the distribution. Find—

(i) the distribution of Y

(ii) $E(Y)$

(b) (i) Mention the chief characteristics of normal distribution. 5

(ii) Let X be a one-parameter exponential distribution with parameter λ. Then for every constant a $(a > 0)$, prove that $P(X \le x + a | X \ge a) = P(X \le x)$ for all $a > 0, \ x > 0$. 5

(c) If X be a non-negative integer valued random variable satisfying

$$P(X > i + j | X > i) = P(X \ge j)$$

for any two positive integers i and j, then show that X must have a geometric distribution.

(d) (i) Show that if the random variables X and Y are iid binomial variables with parameters (n, P) and (m, P) respectively, then the conditional distribution of $\frac{X}{X+Y}$ is hypergeometric. 4

(ii) Prove that for negative binomial distribution

$$P(x) = \binom{k+x-1}{x} q^x P^k; x = 0, 1, 2, \cdots$$

$$\mu_{r+1} = q\left(\frac{d\mu_r}{dq} + \frac{rk}{P^2}\mu_{r-1}\right) \quad 6$$

(e) (i) Let $X \sim \beta_1(m, n)$ and $Y \sim \gamma(\lambda, m+n)$ be independent random variables, $(m, n, \lambda) > 0$. Find a p.d.f. for XY and identify the distribution. 5

(ii) If X follows normal distribution with mean μ and standard deviation σ, find the distribution of $Y = a + bx$. 5

(f) (i) Let X_i $(i = 1, 2, \cdots, n)$ is iid random variable having Weibull distribution. Show that the variable $Y = \min(X_1, X_2, \cdots, X_n)$ also has Weibull distribution. 4

(ii) If X has a Poisson distribution with parameter λ, show that the distribution function of X is given by

$$F(x) = \frac{1}{\Gamma(x+1)} \int_{\lambda}^{\infty} e^{-t}\, t^{x}\, dt;\ x = 0, 1, 2, \cdots$$ 6

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