2017

STATISTICS

(Major)

Paper : 4.1

(Mathematical Methods—III and OR—I)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following as directed : 1×7=7

(a) A feasible solution is always basic.

(State True or False)

(b) What is eigen vector?

(c) Are the vectors $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 10 \end{pmatrix}$ independent?

(d) What is artificial variable?

(e) Define transportation problem.

(f) The characteristic roots of a real —— matrix are all real.

(Fill in the blank)

(g) What do you mean by linearly independent set of vectors?

2. Answer the following questions : 2×4=8

(a) Find the convex hull of 3 points

$a_1 = (6, 6)$, $a_2 = (9, 12)$, $a_3 = (3, 9)$

(b) Write the limitations of graphical method of solution.

(c) When does an LPP have an unbounded solution?

(d) Express the following LPP in standard form :

Minimize $Z = x_1 - 2x_2 + x_3$

Subject to

$$2x_1 + 3x_2 + 4x_3 \geq -4$$
$$3x_1 + 5x_2 + 2x_3 \geq 7$$

$x_1 \geq 0$, $x_2 \geq 0$ and x_3 is unrestricted in sign.

3. Answer any *three* of the following questions : 5×3=15

(a) Express (1, 2, 3) as a linear combination of (1, 1, 1), (2, −1, 1) and (1, −2, 5) in $V_3(R)$.

(b) Define hyperplane. Prove that hyperplane is a convex set.

(c) Find the characteristic roots of the matrix

$$A = \begin{pmatrix} 5 & 2 \\ 2 & 5 \end{pmatrix}$$

and verify that $|A| =$ product of the characteristic roots and tr $A =$ sum of the characteristic roots of A.

(d) State and explain the standard form of LPP.

(e) Find the necessary and sufficient conditions that if λ is a characteristic root of a matrix A, there exists a non-zero vector X such that $AX = \lambda X$.

4. Answer any *three* of the following questions : 10×3=30

(a) *(i)* Prove that in an Euclidean space E^n we cannot have more than n linearly independent vectors. 5

(ii) Show that a necessary and sufficient condition for the existence of a feasible solution to an $m \times n$ transportation problem is

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

where a_i and b_j denote the availability and requirement at ith origin and jth destination respectively. 5

(b) (i) Solve the following LPP graphically : 5

Maximize $Z = 5x_1 + 3x_2$

Subject to the constraints

$$3x_1 + 5x_2 \le 15$$
$$5x_1 + 2x_2 \le 10$$
and $$x_1, x_2 \ge 0$$

(ii) If α is a characteristic root of a non-singular matrix A, then prove that $\frac{|A|}{\alpha}$ is a characteristic root of Adj. A. 5

(c) What do you mean by slack and surplus variables? Solve the following LPP by simplex method : 3+7=10

Maximize $Z = 3x_1 + 5x_2$

Subject to the constraints

$$3x_1 + 2x_2 \le 18$$
$$x_1 \le 4$$
$$x_2 \le 6$$
and $x_1, x_2 \ge 0$

(d) State Cayley-Hamilton theorem. Use it to express $2A^5 - 3A^4 + A^2 - 4I$ as a linear polynomial in A, when

$$A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$ 2+8=10

(e) Show that the collection of all feasible solutions to LPP constitutes a convex set whose extreme points corresponding to the basic feasible solution. 10

(f) Determine the eigenvalues and eigen vectors of the matrix

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

5+5=10

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