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STATISTICS

(Major)

Paper : 4.2

(Descriptive Statistics—II & Probability—II)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following as directed : 1×7=7

(a) If a sequence of constants $a_n \to a$ as $n \to \infty$, then regarding the constant as a random variable having one-point distribution at that point, we can say that $a_n \xrightarrow{P}$ —— as $n \to \infty$.

(Fill in the blank)

(b) Like moment-generating function, characteristic function also generates moments.

(State True or False)

(c) The 95% confidence limits of probability of success (p) is given by ——.

(Fill in the blank)

(d) A continuous time stochastic process may have a discrete or continuous state space.

(State True or False)

(e) A random process that is not stationary in any sense is called an —— process.

(Fill in the blank)

(f) The formula of standard error for the difference of two standard deviations $(s_1 - s_2)$ is ——.

(Fill in the blank)

(g) If the characteristic function of a distribution is given, the distribution can be uniquely determined by the theorem known as ——.

(Fill in the blank)

2. Answer the following questions in brief : 2×4=8

(a) Write the conditions for the existence of weak laws of large numbers.

(b) What are different states of Markov chain?

(c) Write the uses of standard error.

(d) State Levy-Lindeberg theorem.

3. Answer any *three* questions of the following : 5×3=15

(a) Examine whether the weak law of large numbers holds good for the sequence X_n of independent random variables, where

$$P(X_n = \frac{1}{\sqrt{n}}) = \frac{2}{3} \text{ and } P(X_n = -\frac{1}{\sqrt{n}}) = \frac{1}{3}$$

(b) How a large sample must be taken in order that the probability will be at least 0·95 and X_n will be within 0·5 of μ (μ is unknown and $\sigma = 1$)?

(c) Define standard error. Show that the SE of mean of a random sample of size n from a population with variance σ^2 is $\frac{\sigma}{\sqrt{n}}$.

(d) Patients arrive randomly and independently at a doctor's chamber from 8 AM at an average rate of one in five minutes. The waiting room can hold only 12 persons. What is the probability that the room will be full, when the doctor arrives at 9 AM?

(e) An ambulance service claims that it takes on an average 8·9 minutes to reach its destination in emergency calls.

To check on this claim, the agency which licenses ambulance services has timed on 50 emergency calls, getting a mean of 9·3 minutes with a standard deviation 1·6 minutes. What can they conclude at the level of significance $\alpha = 0 \cdot 05$?

(f) Use Tchebycheff's inequality to show that for $n > 36$, the probability that in n throws of a fair dice, the number of sixes lies between $\frac{1}{6}n - \sqrt{n}$ and $\frac{1}{6} + \sqrt{n}$ is at least $\frac{31}{36}$.

4. Answer any *three* of the following questions : 10×3=30

(a) A test of the strength of a wire consists of bending and unbending until it breaks. Considering bending and unbending as two operations, let X denote that random variable corresponding to the number of operations necessary to break the wire. If $P(x = r) = (1 - P)P^{r-1}$, $r = 1, 2, \cdots$ and $0 < P < 1$, find the probability generating function of x.

Or

State and prove the central limit theorem.

(b) Define the following with examples (any *two*) : 5×2=10

(i) Transition probability matrix

(ii) Characteristics of stochastic process

(iii) Classification of states of Markov chain

(c) In a year, there are 956 births in a town *A* of which 52·5% are males. While in towns *A* and *B* combined, this proportion in a total of 1406 births was 0·496. Is there any significant difference in the proportion of male births in the two towns?

(d) As an application of CLT, show that if E is such that $P(|\bar{x}-\mu|<E)>0\cdot95$, then the minimum sample size n is given by $n=1\cdot96\frac{\sigma^2}{E^2}$, where μ and σ^2 are the mean and variance respectively in the population and $\bar{x}$ is the mean of the random sample.

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