2017

STATISTICS

(Major)

Paper : 5.1

(Sampling Distribution and Statistical Interference—I)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following as directed : $1 \times 7 = 7$

(a) State the invariance property of sufficient estimator.

(b) Chi-square distribution is used for the test of

(i) equality of two population variances

(ii) independence of attributes

(iii) equality of several population means

(iv) All of the above

(Choose the correct answer)

(c) State Cramer-Rao lower bound for variance of an estimator.

(d) The m.g.f. of *t*-distribution always exists. (State True or False)

(e) What do you mean by 'asymptotic unbiasedness'?

(f) Student's *t*-distribution was given by

(i) G. W. Snedecor

(ii) R. A. Fisher

(iii) W. S. Gosset

(iv) None of them

(Choose the correct answer)

(g) If the density function $f(x, \theta)$ is continuous and monotonic in θ in interval containing the value θ_0 and for all x_1, then the M. L. estimator $\hat{\theta}$ is ____. (Fill in the blank)

2. Answer the following questions : 2×4=8

(a) Prove that for large d.f., *t*-distribution tends to standard normal distribution.

(b) Define order statistics. State one application of order statistics.

(c) Estimate θ for the distribution

$$f(x,\theta) = \frac{2}{\theta^2}(\theta - x),\ 0 < x < \theta$$

for sample of size one.

(d) Prove with usual notations

$$S.E.(s^2) = \sigma^2\sqrt{\frac{2}{n}}$$

3. Answer any *three* questions : 5×3=15

(a) If X_1 and X_2 are two independent χ^2-variates with n_1 and n_2 degrees of freedom (d.f.) respectively, then prove that

$$\frac{X_1}{X_2} \text{ is a } \beta_2\left(\frac{n_1}{2}, \frac{n_2}{2}\right) \text{ variate.}$$

(b) Prove that if a sufficient estimator exists, then it is a function of the maximum likelihood estimator (MLE).

(c) *(i)* Show that the square of a *t*-variate with n d.f. is distributed as *F*-statistic with $(1, n)$ d.f.

(ii) If X is an *F*-variate with $(2, n)$ d.f. where $n \geq 2$, then show that

$$P(F \geq k) = \left(1 + \frac{2}{n}K\right)^{-\frac{n}{2}}$$

(d) Suppose $x_1, x_2, ..., x_n$ are observations on Bernoulli variate assuming the values 1 and 0 with respective probabilities θ and $(1-\theta)$. Show that $\frac{z(z-1)}{n(n-1)}$ is an unbiased estimator of θ^2

$$\text{where, } z = \sum_{i=1}^{n} x_i$$

(e) Show that $\bar{x} = \sum_{i=1}^{n} \frac{x_i}{n}$ in random sampling from

$$f(x, \theta) = \begin{cases} \frac{1}{\theta} e^{-\frac{x}{\theta}}, & 0 < x < \infty \\ 0 & , \text{ elsewhere} \end{cases}$$

is MVB estimator of θ and has variance $\frac{\theta^2}{n}$.

4. Answer any *three* questions : 10×3=30

(a) Define minimum variance unbiased estimator (MVUE). Prove that an MVUE is unique in the sense that if T_1 and T_2 are MVUEs for a parameter θ, then

$$T_1 = T_2 \text{ almost surely.}$$

(b) Obtain the joint probability density function of r th and s th order statistics.

(c) *(i)* State three applications of student's t-test.

(ii) A random variable takes the values 0, 1, 2 with respective probabilities $\frac{\theta}{4n}+\frac{1}{2}\left(1-\frac{\theta}{N}\right)$, $\frac{\theta}{2N}+\frac{\alpha}{2}\left(1-\frac{\theta}{N}\right)$ and $\frac{\theta}{4n}+\frac{1-\alpha}{2}\left(1-\frac{\theta}{N}\right)$

where N is a known number and θ and α are unknown parameters. If 75 independent observations on X yielded the values 0, 1, 2 with frequencies 27, 38, 10 respectively, estimate the parameters θ and α by the method of moments. 3+7=10

(d) Define F-statistics and derive its distribution.

(e) A sample of size n is drawn from each of the four normal populations which has the same variance σ^2. The means of the four populations are $a+b+c$, $a+b-c$, $a-b+c$ and $a-b-c$ respectively. What are the MLEs for a, b, c and σ^2?

(f) Derive Fisher's t-distribution. Show that student's t-variate may be regarded as a particular case of Fisher's t-variate.

7+3=10

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